

Finite Math - Spring 2017

Lecture Notes - 2/10/2017

HOMEWORK

- Section 3.1 - 55, 58, 63, 71, 80, 81

SECTION 3.1 - SIMPLE INTEREST

We can also use the formulas to predict what interest rate we need or how much principal to take out/deposit.

Example 1. *You're looking to invest \$5,000 and make \$100 in interest after 10 weeks. What annual rate on your investment will you need to accomplish this?*

Solution. *$I = 100$ and the time is $t = \frac{10}{52}$ since there is 52 weeks in a year, so plugging all this into the formula gives*

$$100 = 5000r \left(\frac{10}{52} \right)$$

and we need to solve for r .

$$\begin{aligned} 100 &= 5000r \left(\frac{10}{52} \right) \\ \implies r &= \frac{100}{5000 \left(\frac{10}{52} \right)} \\ &= 0.104 \end{aligned}$$

So we would need an annual rate of 10.4% to make \$100 in interest after 10 weeks.

Example 2. *You invest \$4,000 at an annual rate of 3.9%. How long will it take for the investment to be worth \$5,000? Give your answer in years, correct to 2 decimal places.*

Solution. 6.41 years

One often uses a brokerage firm when making investments, many of which charge you a fee based on the transaction amount (principle) when both buying AND selling stocks.

Example 3. *Suppose a brokerage firm uses the following commission schedule*

<i>Principal</i>	<i>Commission</i>
<i>Under \$3,000</i>	<i>\$25 + 1.8% of principal</i>
<i>\$3,000 - \$10,000</i>	<i>\$37 + 1.4% of principal</i>
<i>Over \$10,000</i>	<i>\$107 + 0.7% of principal</i>

An investor purchases 450 shares of a stock at \$21.40 per share, keeps the stock for 26 weeks, then sells the stock for \$24.60 per share. What was the annual interest rate earned on the investment?

Solution. To purchase 450 shares will cost $\$21.40(450) = \$9,630$. This falls into the second fee range of the commission schedule, so the transaction fee will be

$$\$37 + 0.014(\$9630) = \$171.82.$$

Thus, the total initial investment is

$$\$9,630 + \$171.82 = \$9,801.82.$$

Next, the investor sells the stock for

$$\$24.60(450) = \$11,070$$

This falls into the third fee range on the schedule, so the commission is

$$\$107 + 0.007(\$11,070) = \$184.49.$$

Thus, the net return on the investment is

$$\$11,070 - \$184.49 = \$10,885.51.$$

Now, using the total investment as the principal and the net return as the future value, we can use the future value formula to figure out the annual interest rate earned. $P = 9801.82$, $A = 10885.51$, the time elapsed was 26 weeks, and there are 52 weeks in a year, so $t = \frac{26}{52} = 0.5$.

$$\begin{aligned} 10885.51 &= 9801.82(1 + 0.5r) \\ &= 9801.82 + 4900.91r \\ \implies 1083.69 &= 4900.91r \\ \implies r &= \frac{1083.69}{4900.91} \approx 0.22112 \end{aligned}$$

So the interest rate earned was 22.112%.

Example 4. Suppose a brokerage firm uses the following commission schedule

Principal	Commission
Under \$3,000	\$32+1.8% of principal
\$3,000 - \$10,000	\$56+1% of principal
Over \$10,000	\$106+0.5% of principal

An investor purchases 75 shares of a stock at \$37.90 per share, keeps the stock for 150 days, then sells the stock for \$41.20 per share. What was the annual interest rate earned on the investment? (Again, assume a 360-day year.)

Solution. 6.352%

Average Daily Balance. A common method for calculating interest on a credit card is to use the *average daily balance method*. As the name suggests, the average daily balance is computed, then the interest is computed on that.

Example 5. A credit card has an annual interest rate of 19.99% and interest is calculated using the average daily balance method. If the starting balance of a 30-day billing cycle is \$523.18 and purchases of \$147.98 and \$36.27 are posted on days 12 and 25, respectively, and a payment of \$200 is credited on day 17, what will be the balance on the card at the start of the next billing cycle?

Solution. We must figure out what the balance is on each day of the month. At the end of day 1, the balance is \$523.18. The first transaction happens on day 12, which is a purchase of \$147.98, making the balance \$671.16. The next transaction is on day 17, a payment of \$200, making the balance \$471.16. The next, and final, transaction is on day 25 which is a purchase of \$36.27, making the balance \$507.43. It helps to make a chart of this data

Day 1-11:	\$523.18	(11 days)
Day 12-16:	\$671.16	(5 days)
Day 17-24:	\$471.16	(8 days)
Day 25-30:	\$507.43	(6 days)

To find the average daily balance, we can take the sum of the balance at the end of each day, then divide by the number of days.

$$SUM = 11(523.18) + 5(671.61) + 8(471.16) + 6(507.43) = \$15,924.64$$

Dividing this number by 30 gives the average daily balance

$$ADB = \frac{SUM}{30} = \$530.82.$$

We can use the formula for interest to figure out the interest incurred (assuming 360 days in a year), $t = \frac{30}{360} = \frac{1}{12}$

$$I = Prt = (530.90)(0.1999) \left(\frac{1}{12} \right) = \$8.84.$$

To find the balance at the start of the next billing cycle, we add this interest to the remaining balance at the end of the last cycle:

$$\text{New Balance} = \underbrace{\$507.43}_{\text{Day 30 balance}} + \underbrace{\$8.84}_{\text{Interest}} = \$516.27$$